**PHYSICS**

# DPP

DAILY PRACTICE PROBLEMS

# DPP No. 69

**Total Marks : 20**

**Max. Time : 22 min.**

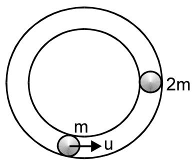
**Topics : Rigid Body Dynamics, Center of Mass, Circular Motion**

| Type of Questions | | M.M., Min. |
|---|---|---|
| Single choice Objective ('–1' negative marking) Q.1 to Q.3 | (3 marks, 3 min.) | [3, 3] |
| Subjective Questions ('–1' negative marking) Q.4 to Q.5 | (4 marks, 5 min.) | [8, 10] |
| Comprehension ('–1' negative marking) Q.6 to Q.8 | (3 marks, 3 min.) | [9, 9] |

**1.** A uniform disk of mass 300kg is rotating freely about a vertical axis through its centre with constant angular velocity $\omega$. A boy of mass 30kg starts from the centre and moves along a radius to the edge of the disk. The angular velocity of the disk now is

(A) $\frac{\omega_0}{6}$ (B) $\frac{\omega_0}{5}$ (C) $\frac{4\omega_0}{5}$ (D) $\frac{5\omega_0}{6}$

**2.** Two masses ' m ' and ' 2 m ' are placed in fixed horizontal circular smooth hollow tube as shown. The mass ' m ' is moving with speed 'u' and the mass ' 2 m ' is stationary. After their first collision, the time elapsed for next collision. (coefficient of restitution e = 1/2)



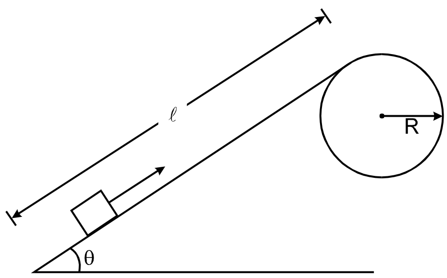

(A) $\frac{2\pi r}{u}$ (B) $\frac{4\pi r}{u}$

(C) $\frac{3\pi r}{u}$ (D) $\frac{12\pi r}{u}$

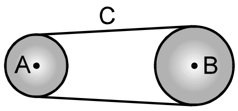
**3.** A solid homogeneous cylinder of height h and base radius r is kept vertically on a conveyer belt moving horizontally with an increasing velocity $v = a + bt^2$. If the cylinder is not allowed to slip then the time when the cylinder is about to topple, will be equal to

(A) $\frac{rg}{bh}$ (B) $\frac{2rg}{bh}$ (C) $\frac{2bg}{rh}$ (D) $\frac{rg}{2bh}$

**4.** Figure shows a smooth track which consists of a straight inclined part of length $\ell$ joining smoothly with the circular part. A particle of mass m is projected up the inlcine from its bottom. (a) Find the minimum projection - speed $v_0$ for which the particle reaches the top of the track. (b) Assuming that the projection - speed is $2v_0$ and that the block does not lose contact with the track before reaching its top, find the force acting on it when it reaches the top. (c) Assuming that the projection-speed is only slightly greater than $v_0$, where will the block lose contact with the track?

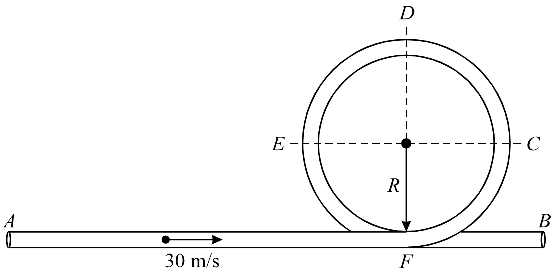


**5.** Wheel A of radius $r_A$ = 10cm is coupled by a belt C to another wheel of radius $r_B$ = 25 cm as in the figure. The wheels are free to rotate and the belt does not slip. At time t = 0 wheel A increases it's angular speed from rest at a uniform rate of $\pi/2$ rad/sec$^2$. Find the time in which wheel B attains a speed of 100 rpm **[Hint: $v_A = v_B$]**




**COMPREHENSION**

A smooth horizontal fixed pipe is bent in the form of a vertical circle of radius 20 m as shown in figure. A small glass ball is thrown in horizontal portion of pipe at speed 30 m/s as shown from end *A*. (Take *g* = 10 m/s$^2$)




**6.** Which of the following statement is/are correct :
(i) ball will not come out from end *B*.
(ii) ball will come out from end *B*.
(iii) At point D speed of ball will be just more than zero.
(iv) At point E and C the ball will have same speed.
(A) only (i) (B) (ii) and (iv) (C) (ii), (iii) and (iv) (D) only (ii)

**7.** At which angle from vertical from bottom most point *F*. The normal reaction on ball due to pipe will change its direction (in terms of radially outwards and inwards) :

(A) $\theta = 180°$ (B) $\theta = \cos^{-1}\left(-\frac{2}{3}\right)$ (C) $\theta = \cos^{-1}\left(-\frac{5}{6}\right)$ (D) None of these

**8.** With what speed ball will come out from point *B* :

(A) 30 m/s (B) $20\sqrt{2}$ m/s (C) $10\sqrt{5}$ m/s (D) None of these

# Answers Key

## DPP NO. - 69

**1.** (D) **2.** (B) **3.** (A)

**4.** (a) $\sqrt{2g[R(1-\cos\theta)+\ell\sin\theta]}$

(b) $6\,mg\left(1-\cos\theta+\frac{\ell}{R}\sin\theta\right)$

(c) The radius through the particle makes an angle $\cos^{-1}(2/3)$ with the vertical **5.** 50/3 sec.

**6.** (B) **7.** (C) **8.** (A)

# Hint & Solutions

## DPP NO. - 69

**1.** As $\Sigma\tau = 0$, angular momentum remains conseved :

$$\therefore \quad L = \left(0 + \frac{300R^2}{2}\right)$$

$$\omega_0 = \left(\frac{300R^2}{2} + 30R^2\right).\omega$$

$\Rightarrow$ $150\ \omega_0 = 180\ \omega$

$\Rightarrow$ $\omega = 5/6\ \omega_0$ **Ans.**

**2.** **(B)** Let the speeds of balls of mass m and 2m after collision be $v_1$ and $v_2$ as shown in figure. Applying conservation of momentum

$mv_1 + 2mv_2 = mu$ and $-v_1 + v_2 = -$

solving we get $v_1 = 0$ and $v_2 = \frac{u}{2}$

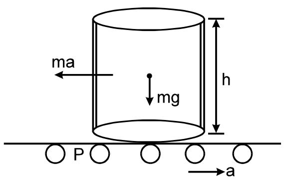
Hence the ball of mass m comes to rest and ball of mass 2m moves with speed $-$.

$$t = \frac{2\pi r}{u/2} = \frac{4\pi r}{u}$$

**3.** WRT to belt, pseudo force ma acts on cylinder at COM as shown about to cylinder will be just about to topple when torque to weight w.r.t. P.

$$\frac{dv}{dt} = a = 2bt$$

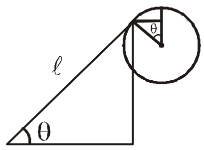
$$m.2bt\ .\ \frac{h}{2} = mg.r$$




$t = \frac{rg}{bh}$ **Ans.: gr/bh**

**4.** (a) $\frac{1}{2}mv_0^2 = mg\,\ell\sin\theta + mgR(1-\cos\theta)$

$v_0 = \sqrt{2gR(1-\cos\theta) + 2g\ell\sin\theta}$




(b) C.O.E.

$= \frac{1}{2}m(2v_0)^2 - mg\,\ell\sin\theta - mg\,R(1-\cos\theta) = \frac{1}{2}mv^2$

$= 2mv_0^2 - mg\ell\sin\theta - mgR(1-\cos\theta) = \frac{1}{2}mv^2$

$= 4mgR\,(1-\cos\theta) + 4\,mg\,\ell\sin\theta - mg\ell\sin\theta$

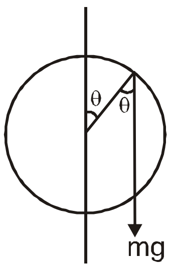
$-mgR(1-\cos\theta) = \frac{1}{2}mv^2$

$= 6mgR(1-\cos\theta) + 6mg\,\ell\sin\theta = mv^2$

$N = 6mg(1-\cos\theta) + 6mg\frac{\ell}{R}\sin\theta$

$= 6mg[(1-\cos\theta) + \frac{\ell}{R}\sin\theta].$

(c)




$mg\cos\theta = \frac{mv^2}{R}$

$= \frac{1}{2}mv^2 = \frac{1}{2}mg\,R\cos\theta$

$= mgR(1-\cos\theta) = \frac{1}{2}mg\,R\cos\theta$

$\cos\theta = \frac{2}{3} \quad \theta = \cos^{-1}\left(\frac{2}{3}\right).$

**5.** For no slipping condition

$r_A \alpha_A = r_B \alpha_B$

$$\Rightarrow \quad \alpha_B = \frac{r_A}{r_B}\alpha_A = \frac{10}{25} \times \frac{\pi}{2} = \frac{\pi}{5} \text{ rad/s}^2$$

$$\omega_B = \frac{2\pi \times 100}{60} = \frac{10\pi}{3} \text{ rad/s}$$

$$\omega_B = \omega_{BO} + \alpha_B t$$

$$\frac{10\pi}{3} = 0 + \frac{\pi}{5} t \Rightarrow t = \frac{50}{3} \text{ sec}$$

**6. to 8** In the given situation if the speed becomes zero at the highest point then also the particle can complete the circle as there is no chance for the particle to loose contact in this case.

$u_{min}$ = minimum speed required to complete vertical circle

$$= \sqrt{4gR} = \sqrt{4 \times 10 \times 20} = \sqrt{800} \text{ m/s}$$

$30 \text{ m/s} > \sqrt{800}$

so it can easily complete the vertical circle

Now, for point C

$k_f + p_f = p_i + k_i$

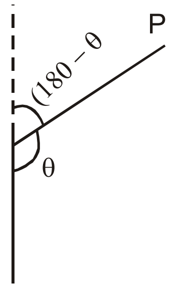
$$\frac{1}{2} m v_c^2 + mgh_c = 0 + \frac{1}{2} m (30)^2$$

$$v_c^2 = (30)^2 - 2gh_c$$

As $h_c = h_E = R$ ; heights of points C & E from reference

so $V_E = V_C$

**7.**




$$mg \cos (180 - \theta) = \frac{mv^2}{\ell} \qquad \text{..... (1)}$$

Applying W – E theoren between points F & P :

$$\frac{1}{2} mu^2 = \frac{1}{2} mv^2 + mg\ell(1 - \cos\theta)$$

$$v^2 = u^2 - 2g\ell(1 - \cos\theta) \qquad \text{..... (2)}$$

on putting the value of $v^2$ from (2) in (1)

$$mg \cos(180 - \theta) = \frac{m}{\ell}(u^2 - 2gl(1 - \cos\theta))]$$

$$-g\ell \cos\theta = u^2 - 2g\ell + 2g\ell \cos\theta$$

$$-3g\ell \cos\theta = 900 - 2 \times 10 \times 20$$

$$\cos\theta = -\frac{500}{3g\ell} = -\frac{500}{600}$$

$$\cos\theta = -5/6$$

**8.** As there will be no energy dissipation, it will come out at the same speed at which it enters.